

EXERCISES: SESSION 5

1. If the coordinates of some vector $v \in \mathbb{R}^3$ are $(1, 1, 1)$ w.r.t. the basis

$$\mathcal{B} = \{(1, 0, 0), (0, 1, 1), (1, 0, 1)\},$$

what are the coordinates of v in the canonical basis?

2. Give the coordinates of the vector $v = (1, -2, 1)$ in coordinates of the basis

$$\mathcal{B} = \{(1, 0, 0), (0, 1, 1), (1, 0, 1)\}.$$

3. Let $\mathcal{B}_1 = \{(1, 0, 0), (0, 1, 1), (1, 0, 1)\}$ and $\mathcal{B}_2 = \{(1, 1, 0), (1, 0, 1), (0, 1, 1)\}$ be two bases of $\mathbb{R}^3$. What are the coordinates of a vector v in the basis $\mathcal{B}_1$ if we know that its coordinates in $\mathcal{B}_2$ are $(0, -1, 1)$?

4. Find the matrix of the linear transformation $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ that produces a reflection with respect to the line with positive slope that cuts the x-axis with an angle of $\theta = \pi/3$ radians.

5. Find the matrix of the linear transformation $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ that produces an orthogonal projection onto the line with positive slope that cuts the x-axis with an angle of $\theta = \pi/3$ radians.

6. Find the matrix of the linear transformation $\mathbb{R}^3 \rightarrow \mathbb{R}^3$ that produces a reflection with respect to the plane spanned by the vectors $(1, 1, 0)$ and $(1, 0, -1)$.