EXERCISES: SESSION 7

Exercise 1. Given the 2×2 matrix:

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

a) Compute the caracteristic polynomial of A: $Q(t) = \det(A - tI)$
b) Compute the eigenvalues of A as the roots of $Q(t)$

Exercise 2. Given the 2×2 matrix:

$$A = \begin{bmatrix} 8 & 3 \\ 2 & 7 \end{bmatrix}$$

a) Compute the caracteristic polynomial of A: $Q(t) = \det(A - tI)$
b) Compute the eigenvalues of A as the roots of $Q(t)$
c) For each eigenvalue, say λ, compute the corresponding eigenvector(s) as non-zero elements of $N(A - \lambda I)$

Exercise 3. Given the 2×2 matrices:

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

a) Are the eigenvalues of AB equal to the eigenvalues of A times the eigenvalues of B?
b) Are the eigenvalues of AB equal to the eigenvalues of BA

Exercise 4. Given the following 2×2 Markov matrices:

$$A = \begin{bmatrix} 0.6 & 0.2 \\ 0.4 & 0.8 \end{bmatrix} \quad A^{\infty} = \begin{bmatrix} 1/3 & 1/3 \\ 2/3 & 2/3 \end{bmatrix}$$

a) Find the eigenvalues and eigenvectors of A
b) Find the eigenvalues and eigenvectors of A^{∞}
c) Explain from those answers why A^{100} is close to A^{∞}

Exercise 5. Given a square matrix A:

a) Prove that $\det(A - \lambda I) = \det(A^t - \lambda I)$
b) The eigenvalues of A equal the eigenvalues of A^t. Why?
c) But the eigenvectors of A may not equal the eigenvectors of A^t. Can you provide an example?

Exercise 6. Supose that a square matrix A has eigenvalues 0, 3, 5 with linearly independent eigenvectors u, v, w.

a) Give a basis of the null space $N(A)$
b) Give a basis of the columns space $C(A)$
c) Find a particular solution to the system $Ax = v + w$. Find all solutions.
d) Why does the system $Ax = u$ have no solution?

Exercise 7. Given the square matrix:

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$$

a) Factor A as CDC^{-1} with D being diagonal.

Exercise 8. Given the square matrix A:

$$A = \begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix}$$

a) Factor A as CDC^{-1} with D being diagonal.