

EXERCISES: SESSIONS 11-12

1. FUNCTIONS IN ONE VARIABLE

1. Compute the derivative of the following functions:

- (1) $f(x) = \sqrt{x}$
- (2) $f(x) = \log(x)$ (natural logarithm)
- (3) $f(x) = \log(1 + e^x)$

2. Consider the function $\text{ReLU}(x) = \max(0, x)$. Is ReLU differentiable at $x = 0$?

3. Given the polynomial function $P(x) = 3x^2 - x - 1$, find the coefficients $a_0, a_1, a_2 \in \mathbb{R}$ that satisfy $P(x) = a_0 + a_1(x - 2) + a_2(x - 2)^2$.

4. Given any degree 2 polynomial $P(x)$ and a point x_0 , describe a general procedure to find $a_0, a_1, a_2 \in \mathbb{R}$ such that

$$P(x) = a_0 + a_1(x - 2) + a_2(x - 2)^2.$$

5. Compute the local Taylor approximation of degree 4 at $x_0 = 0$ of the functions:

- (1) $f(x) = \sin(x)$
- (2) $f(x) = \cos(x)$
- (3) $f(x) = \log(1 - x)$
- (4) $f(x) = \log(1 + e^x)$
- (5) $f(x) = e^x/(1 + e^x)$